(1) Objectives for today's sharing session
(2) Quick recap on heuristics skills
(3) Hands-on Session : Let's learn how to infer
(4) Summary and Q \& A

## Objective

1) To make connections with problem-solving strategies (heuristics) to solve word problems that involve mathematical reasoning and spatial visualization.
2) To infer and solve word problems.



## PSLE 2023 Paper 1 Q27

 10050000050
## Grouping question / Find the number of sets



Infer that one group = 10 eggs +2 eggs free = 12 eggs in total
Mrs Lim took home 120 eggs.
She paid $\$ 4.80$ less with the special offer.
What was the price of 10 eggs without the special offer?

$$
\begin{aligned}
\text { Number of groups } & =120 \div(10+2) \\
& =10
\end{aligned}
$$

Total number of free eggs $=10 \times 2$

$$
=20 \rightarrow \text { Infer that } 20 \text { eggs }=\$ 4.80 \text { paid lesser. }
$$

$$
\begin{aligned}
\text { Cost of } 1 \text { egg } & =\$ 4.80 \div 20 \\
& =\$ 0.24
\end{aligned}
$$

Cost of 10 eggs $=10 \times \$ 0.24$

$$
=\$ 2.40
$$

## PSLE 2023 Paper 1 Q29

 $[500000000$Gopal had 3 identical tins of paint that were completely filled.

He poured 760 ml out from each tin.
The total amount of paint left in the 3 tins after
pouring was equal to the amount of paint in 1
tin at first.
What was the amount of paint in each tin at first?

## Equal Concept $\rightarrow$ Draw a model

|  | 760 ml |
| :---: | :---: |
|  | 760 ml |
|  | 760 ml |

Poured out 760 ml from each tin


Hence,
2 units $=760$
1 unit $=760 \div 2$

$$
=380
$$

1 tin of paint (at first) $=380+760$

$$
=1140
$$

or 1 tin of paint (at first) $=380 \times 3$

$$
=1140
$$

## PSLE 2023 Paper 2 Q15

 000000000Meiyi had some books.
She donated $\frac{3}{8}$ of the books and gave away
another 24 books.
She had $\frac{4}{7}$ of the books left and she packed them into 20 hoxes.
Some boxes contained 10 books while the rest contained 18 books.
(a) How many books were packed inte the 20 boxes?
(b) How many boxes contained 18 books?

Infer total 8 units

| 3 units | 5 units <br> gave away |
| :---: | :---: |

gave away 24 books

(a) Common multiples of 8 and $7=56$

$$
\begin{aligned}
& \text { Donated } \frac{3}{8}=\frac{21}{56} \quad \text { Left } \frac{4}{7}=\frac{32}{56} \\
& 24 \text { books }=56 u-32 u-21 u \\
& \quad=3 u \\
& 3 u=24 \\
& \begin{aligned}
1 u & =24 \div 3 \\
& =8 \\
32 u & =32
\end{aligned} \\
& =8 \\
& =
\end{aligned}
$$

(b) Assume all 20 boxes contained 10 books, $10 \times 20=200$
Total difference $=256-200$

$$
=56
$$

Each box difference $=18-10$

$$
=8
$$

$56 \div 8=7$ boxes contained 18 books

## SIMILAR QUESTION PSLE 2016 Paper 2 Q14

## $[000000000]$

Suyin baked some pies. She gave $\frac{1}{5}$ of them to her relatives and 30 of them to her friends. She was left with $\frac{2}{3}$ of the pies. She packed these into 18 boxes. Some boxes contained 6 pies while the rest contained 12.
(a) How many pies were packed into the 18 boxes?
(b) How many boxes contained 6 pies?
(a) Common multiples of 3 and $5=15$

$$
\text { gave away } \frac{1}{5}=\frac{3}{15} \quad \text { Left } \frac{2}{3}=\frac{10}{15}
$$

$$
\begin{aligned}
30 \text { pies } & =15 u-10 u-3 u \\
& =2 u
\end{aligned}
$$

$$
2 u=30
$$

$$
1 u=30 \div 2
$$

$$
=15
$$

$$
10 u=10 \times 15
$$

$$
=150
$$

(b) Assume all the pies are packed into boxes of 12, $18 \times 12=216$
Total difference $=216-150$

$$
=66
$$

Each box difference $=12-6$

$$
=6
$$

$66 \div 6=11$ boxes contained 6 pies

## PSLE 2022 Paper 1 Q15

 $[000000000$Matthew pushes two sticks, $A$ and $B$, straight into the ground until the length of each stick that is above the ground is the same.

$\frac{1}{3}$ of $A$ and $\frac{1}{8}$ of $B$ are in the ground.
The length of $A$ in the ground is 30 cm longer than the length of $B$ in the ground.
What is the total length of sticks $A$ and $B$ ?

Infer sticks $A$ and $B$ have equal length above the ground.

Common multiples of 2 and $7=14$
$\mathrm{A}\left(\right.$ above ground) $=\frac{2}{3}=\frac{14}{21}$
$B($ above ground $)=\frac{7}{8}=\frac{14}{16}$
Total units $(A)=21 u$
Total units $(B)=16 u$
Difference $=21 u-16 u$

$$
=5 u
$$

$5 u=30$
$1 u=30 \div 5$
$=6$
Total length $(A+B)=37 \times 6$

$$
=222
$$

Equal concept $\rightarrow$ Draw a model


## PSLE 2021 Paper 2 Q3

 [50025050505]A shelf can be packed from end to end with 30 large books or 45 small books. Kevin already packed the shelf with 3 large books and 23 small books. At most, how many more large books can Kevin pack the shelf with?

Large books: Small books
30
45
$=2: 3$
$=\frac{2}{3} \quad: \quad 1$
$=15 \frac{1}{3}$
23

Space already taken on shelf
$=15 \frac{1}{3}+3$
$=18 \frac{1}{3}$

Number of more large books Kevin can pack the shelf with $=30-18 \frac{1}{3}$

$$
\begin{aligned}
& =11 \frac{2}{3} \\
& \approx 11
\end{aligned}
$$

## PSLE 2023 Paper 1 Q24

 $[000000000$The table shows the number of male and female members in a club in June. The number of female adults is not shown.

| Age Group | Number of members in June |  |
| :---: | :---: | :---: |
|  | Male | Female |
| Youth <br> (Below 20 years) | 15 | 28 |
| Adult <br> (20 to 59 years) | 15 | $?$ |
| Senior Citizen <br> (60 years and above) | 32 | 44 | a) $50 \%$ of all the female members in the club were adults. How many female adults were there in the club?

Infer that all the female members are represented by $100 \%$, so the other $50 \%$ of the female members are youths and senior citizens.

So,
Number of female adults = Number of female youths + Number of senior citizens
Number of female adults $=28+44$

$$
=72
$$

## PSLE 2023 Paper 1 Q24

$[5005000005$
The table shows the number of male and female members in a club in June. The number of female adults is not shown.

| Age Group | Number of members in June |  |
| :---: | :---: | :---: |
|  | Male | Female |
| Youth <br> (Below 20 years) | 15 | 28 |
| Adult <br> (20 to 59 years) | 15 | $?$ |
| Senior Citizen <br> (60 years and above) | 32 | 44 |

$=15+15+32$
$=62$

Total number of members
$=144+62$
$=206$
Percentage of members that are males, at first
$=\frac{62}{206} \times 100 \%$
$\approx 30.1 \%$

Suppose 10 female members left:

Total number of members, after = 206-10
$=196$

Percentage of members that are males, after
$=\frac{62}{196} \times 100 \%$
$\approx 31.6 \%$

## PSLE 2021 Paper 2 Q12

55055355
The figure is made up of three rectangles.
A straight line drawn across the rectangles,
divides the figure into two parts: shaded
and unshaded.
a) The perimeter of the shaded part is 4 cm longer than the perimeter of the unshaded
part. What is the length of $A B$ ?


Observe the diagram. Infer that there are some common lengths of the shaded and unshaded parts.

After comparing the common lengths of the shaded and unshaded parts, infer that the two blue sides add up to the difference of 4 cm .

Length of one blue side $=4 \div 2=2$
Length of $A B=(7+3+3)-2$

$$
=11
$$

## PSLE 2021 Paper 2 Q12

$[5000000000$
The figure is made up of three rectangles.
A straight line drawn across the rectangles, divides the figure into two parts: shaded and unshaded.
b) What is the area of the shaded part?


Observe the diagram. Based on visualisation, infer that the area of the shaded part is obtained by subtracting the area of the unshaded triangle from the area of the three rectangles.

Length of largest rectangle $=7+3+3$

$$
=13
$$

Length of medium rectangle $=3+3$

$$
=6
$$

Area of three rectangles $=(3 \times 2)+(6 \times 4)+(13 \times 8)$

$$
=134
$$

Area of unshaded triangle $=\frac{1}{2} \times(2+4+8) \times 11$

$$
=77
$$

Area of shaded part $=134-77$

$$
=57
$$

## PSLE 2023 Paper 2 Q16

 $[000000000$Figure 1 shows a triangular card $P Q R$ with $P Q=Q R$. A number of such cards were arranged along the four sides of a rectangular board. Figure 2 shows part of the arrangement. A total of 24 pipls were placed at an equal distance of 40 cm apart to hold the cards.
a) Find the length of $A B$.

Infer that the length of $A B$ = length of the rectangle. Find the number of cards along the length.

## PSLE 2023 Paper 2 Q16

 $[500000000$Figure 1 shows a triangular card PQR with $P Q=Q R$. A number of such cards were arranged along the four sides of $a$ rectangular board. Figure 2 shows part of the arrangement. A total of 24 pins were placed at an equal distance of 40 cm apart to hold the cards.
b) Find the total area of the cards used.

## Heuristics: Gaps and Differences

Infer that the area of one card may be found using information from Figure 1.


Figure 1


Infer that the total number of cards used were placed along two lengths and two breadths of the rectangle.

Total number of cards $=8 \times 2+3 \times 2$

$$
=22
$$

Area of one card $=\frac{1}{2} \times 40 \times 20$

$$
=400
$$

Total area $=400 \times 22$

$$
=8800
$$

Ans: $8800 \mathrm{~cm}^{2}$

## PSLE 2016 Paper 2 Q15

$[000000000$

A total of 18 light bulbs are set up at an equal distance apart along three sides $A B, B C$ and $C D$ of rectangular platform. The figure shows part of the set-up. The breadth of the platform is 260 cm . What is the length of the platform?

Infer that 1 length of the rectangular platform does not have light bulbs, so the 18 light bulbs only cover 1 length and 2 breadths.

Based on the diagram, 5 light bulbs cover the breadth of 260 cm .
Infer that there are 4 gaps between 5 light bulbs along the $A B$ and CD.
(Heuristics: Gaps and Differences)
So, the length of one gap $=260 \div 4$

$$
=65
$$

For $A B$ and $C D$,
Number of light bulbs $=5 \times 2$

$$
=10
$$

Remaining light bulbs $=18-10$

$$
=8
$$



For BC,

$$
\begin{aligned}
\text { Number of light bulbs } & =8+1+1 & \text { Length of platform } & =9 \times 65 \\
& =10 & & 585
\end{aligned}
$$

Number of gaps along BC = 10-1

$$
\text { = } 9
$$

## PSLE 2020 Paper 1 Q13

000000000
Figure 1 shows a rectangular tile with a
Recall that the perimeter of a rectangle is the sum of 2 lengths and 2 breadths.
perimeter of 14 cm .
Figure 2 is formed using 5 such tiles.
Figure 1


Find the perimeter of Figure 2.
$\square$


Visualisation: Shift the rectangles to find the total number of lengths and breadths that make up the outline of Figure 2.

2 lengths +2 breadths $=14 \mathrm{~cm}$
1 length +1 breadth $=14 \mathrm{~cm} \div 2$
$=7 \mathrm{~cm}$
Perimeter of figure $2=6$ lengths +6 breadths

$$
\begin{array}{ll}
=6 \text { lengtns }+6 \text { breadths } & \text { (1) } 42 \mathrm{~cm} \\
=6 \times 7 & \text { (2) } 49 \mathrm{~cm} \\
=42 & \text { (3) } 56 \mathrm{~cm} \\
& \text { (4) } 70 \mathrm{~cm}
\end{array}
$$

## PSLE 2010 Paper 1 Q28

 $[000000000$The shaded figure below is formed using 3 squares and 3 equilateral triangles. The length of the straight line $A B$ is 15 cm .

Recall that squares have 4 equal sides and equilateral triangles have 3 equal sides.

Infer that
Line $A B=1$ "green" + 1 "red" + " 1 blue"
Infer that

- Side of small square = Side of small triangle ("blue")
- Side of medium square = Side of medium triangle ("green")
- Side of large square = Side of large triangle ("red")

Perimeter of shaded figure $=5$ sets of blue, green and red sides
$=5 \times$ Line $A B$
$=5 \times 15$
$=75$

## PSLE 2021 Paper 1 Q30

 $[000000000$In Figure 1, the total perimeter of 4 rectangles $R$ and square $S$ is 144 cm . They are joined to form a large square in Figure 2 which has a perimeter of 56 cm .


Find the length of one side of square $S$

From Figure 1,
There are 8 lengths of rectangle $R, 8$ breadths of rectangle $R$ and 4 sides of square $S$.

From Figure 2,
4 lengths of $R+4$ breadths of $R=56 \mathrm{~cm}$
8 lengths of $R+8$ breadths of $R=112 \mathrm{~cm}$
By Elimination method
8 !engths of $R+8$ breadths of $R+4$ sides of square $S=144$
8 lengths of $R+8$ broadths of $R=112 \mathrm{~cm}$
Length of 4 sides of square $S=144 \mathrm{~cm}-112 \mathrm{~cm}$

$$
=32 \mathrm{~cm}
$$

Length of 1 side of square $\mathrm{S}=32 \mathrm{~cm} \div 4$

$$
=8 \mathrm{~cm}
$$

## PSLE 2002 Paper 1

 $[000000000$The figure shows a square of side 14 cm , two semi-circles and a circle.

(a) Find the area of the shaded portions.
(b) Find the perimeter of the shaded part labelled A .
(Take $\pi=\frac{22}{7}$ )

(a) Area of shaded portions
$=$ Area of half square - Area of 2 quarter circles
$=(14 \times 7)-\left(2 \times \frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right)$
$=98-77$
$=21 \mathrm{~cm}^{2}$
(b) Perimeter of shaded part A

$$
\begin{aligned}
& =2 \times \frac{1}{4} \times \frac{22}{7} \times 14 \\
& =22 \mathrm{~cm}
\end{aligned}
$$



Provide opportunities for children to explore maths at home and in the environment

- Learn about shapes
- Discover space and develop metacognition through their five senses


## Real-world math activities:

Using daily activities as problem-solving tasks for children to apply concepts learnt

- Cooking: units of measurements, divide food into equal portions, use of proportions when following recipes
- Organise spaces at home, packing (pattern awareness, ordering, understanding of sets, space and spatial awareness)


To achieve the $21^{\text {st }}$ Century Competencies for Mathematics, we harness opportunities for students to make connections with heuristics strategies and problem solving. To do so, they need to practise the following:

## CRITICAL AND INVENTIVE THINKING

Use different strategies or formulate different mathematical models to solve open-ended or realworld problems.

## ADAPTIVE THINKING

Use various approaches to solve different but related problems.



